NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 08BSMH	LEVEL: 8
COURSE CODE: FAN802S	COURSE NAME: FUNCTIONAL ANALYSIS
SESSION: NOVEMBER 2019	PAPER: THEORY
DURATION: 3 HOURS	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	PROF. G. HEIMBECK
MODERATOR:	PROF. F. MASSAMBA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 4 PAGES (Including this front page)

Question 1 [16 marks]

a) What is an order relation? State the definition. Is it possible to order every set? Give reasons. [5]

b) State Zorn's lemma. [4]

c) Let $(X, \leq)$ be an ordered set which satisfies the hypothesis of Zorn's lemma. Show that every maximal chain in X has a greatest element. [7]

Question 2 [12 marks]

Let V be a $\mathbb{K}$-vector space.

a) What is a basis of V? State the definition. If $V = \{0\}$, does V have a basis? Give reasons. [3]

b) Prove that every minimal spanning set of the vector space V is a basis of V. [5]

c) Does every spanning set of the vector space V contain a minimal spanning set of V? Explain your answer? [4]

Question 3 [14 marks]

Let V be a $\mathbb{K}$-vector space.

a) What is a norm in V? State the definition. [4]

b) Prove that there exists a norm in V. [8]

c) What does one gain from a norm in V? Explain your answer. [2]

Question 4 [14 marks]

Let $(V, \| \ \|)$ be a normed vector space.

a) Let U be a subspace of V. If U is a Banach space, prove that U is closed. [6]

b) Let V be a Banach space and U a closed subspace of V. Prove that U is a Banach space. [6]

c) Are subspaces of finite dimension of V closed? Give reasons. [2]

Question 5 [14 marks]

a) What is a Euclidean vector space? State the definition. [4]

b) Let (V, Φ) be a Euclidean vector space.

i) If R and S are subspaces of V, prove that

$$(R+S)^{\perp} = R^{\perp} \cap S^{\perp}.$$

[4]

ii) If U is a subspace of V, prove that $U^{\perp}$ is closed. [6]

Question 6 [19 marks]

Let (V, Φ) be a unitary vector space of infinite dimension.

a) Prove that there exists a sequence $(a_n)_{\mathbb{N}}$ of orthonormal vectors. [8]

b) Show that $\sum \frac{1}{2^k} a_k$ is absolutely convergent. [3]

c) If $\sum \frac{1}{2^k} a_k$ is convergent, prove that $\sum\limits_{k=1}^{\infty} \frac{1}{2^k} a_k$ does not belong to $[\{a_k | \ k \in \mathbb{N}]$. [8]

Question 7 [11 marks]

Let $(V, \| \ \|)$ be a normed $\mathbb{K}$-vector space.

a) Let U be a subspace of V and $\alpha : U \to \mathbb{K}$ a linear form. If α is continuous, state the definition of $\|\alpha\|$. [2]

b) State the theorem of Hahn-Banach. [4]

c) Let $a \in V - \{0\}$. Show that there exists a closed hyperplane H of V such that $a \notin H$. What is the intersection of all closed hyperplanes of V? Give reasons. [5]

End of the question paper